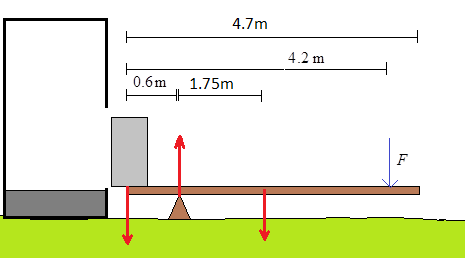
**Homework 7 Solutions Due 3/6**

\* Note, the space provided doesn’t really correlate to the expected amount of work necessary to solve the problem – I’m just trying to keep things tidy.

**Problem 1.** Let’s consider how a lever can reduce the force required to lift an object. Suppose you’re trying to load a large crate into a shipping container. The crate has a mass of 325 kg. The length of the lever you’re using is ℓ = 4.7m, and its mass is m = 13kg.



(a) What is the weight of the crate?

Weight is:



Which is around 700lbs.

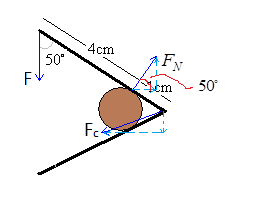
(b) What force F must you apply? Don’t forget to take into account the mass of the board ☺.

The forces acting on the board are shown above. Now we set our axis of rotation to be the fulcrum and apply the rotational N2L.



This is around 110lbs. So definitely doable, especially if you’re standing on it.

**Problem 2.** As another example of how levers can multiply applied force, consider the nut-cracker shown below. You can ignore gravity acting on the upper lever. Let F be 50N.



(a) If you exert a force of F on the ends as shown, what force, FN, is exerted on the nut?

To determine FN, let’s apply rotational N2L to the upper lever, taking our axis of rotation to be the hinge. Then we have,



so we see that the force the nut exerts on the lever (which by N3L is the force the lever exerts on the nut) is 3.83 times the force applied to the lever. This is the usefulness of levers.

(b) What is the magnitude and direction of the contact force the hinge exerts?

Now what of the force that the hinge exerts on the lever (this would be important to know if we don’t want the hinge to break)? We can use N2L for translation to determine this.



And,

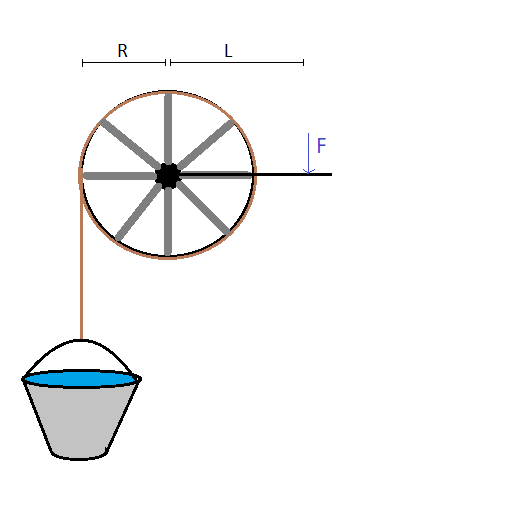
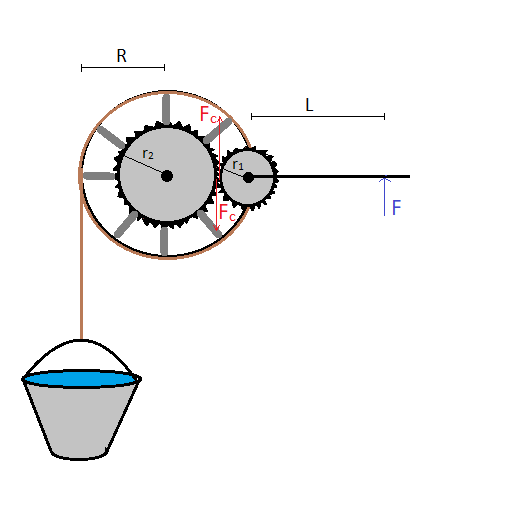


The magnitude and direction of **F**c is



So the force on the hinge will also be much larger than the force applied to the lever, F.

**Problem 3.** Now let’s consider a different application – to how gears can magnify (or diminish) an applied torque. Let the mass of the bucket of water be m = 20kg. Let the radius of the wheel be R = 30cm, and the length of the lever be L = 45cm. Also let the radii of the gears be r2 = 20cm, and r1 = 10cm. And suppose everything is rotating at a constant rate (or not at all).

(a) In the left diagram, symbollically in terms of L and F, what torque does the force F exert on the wheel?

The torque is τ = -LF.

(b) In the left diagram, what force must F be to raise the bucket?

It must counter the torque exerted by the bucket, i.e.,



(c) In the right diagram, symbolically in terms of L and F, what torque does the force F exert on the first gear?

Torque is τ1 = LF, like before, except for the difference in sign.

(d) The first gear exerts a contact force on the second gear, Fc. The second gear will exert the same contact force Fc on the first gear (by N3L). Symbollically, in terms of L, F, and r1, what is Fc?

We can obtain the contact force by applying N2L for rotation to the first gear:



(e) Symbolically, in terms of L, F, r1, and r2, what torque does Fc exert on the second gear then?

The torque exerted on gear 2 is:



(f) Symbolically, in terms of L, F, r1 and r2, what torque is exerted on the wheel then? And how does this compare to your answer in part (a)?

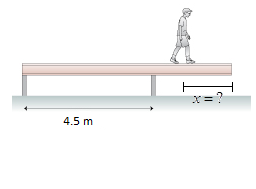
The torque on the 2st gear is the same as that on the wheel. So, τ = -(r2/r1)LF. We see that with the gears present, the torque gets amplified by the ‘gear ratio’, i.e. the ratio of the radii of the two gears. Since r2 > r1, the torque will be amplified.

(g) What force F is required to raise the bucket now?

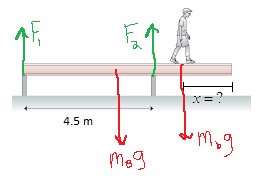
Applying N2L rotation to the wheel again, we have:



**Problem 4**. A 50kg, 7m long beam is supported, but not attached to, the two posts in the figure. A 25kg boy starts walking along the beam. How close can he get to the end before the the beam tips over? Think of what the force the left post will exert on the beam will be, when it starts to tip….



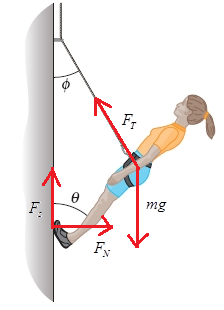
Forces look like this:



When it’s about to tip, F1 = 0. So doing rotational N2L about the F2 reference point we get:



**Problem 5.** In the figure, a climber with a weight of 480 N is held by a belay rope connected to her climbing harness and belay device; the rope is attached at her center of mass. Say φ = 30°, what is the smallest θ can be before her feet start to slip. Let μs = 0.75.



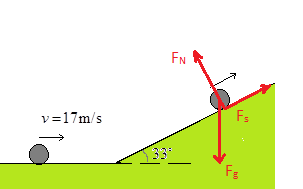
Let ℓ be the distance between her center of mass and her toes. Then rotational N2L about the center of mass says,



when on the verge of slipping Fs is at its maximum value Fs = μsFN. So filling this into our equation above, we have:



**Problem 6.** Suppose you roll a disk up a slope with initial velocity v = 17m/s. When will it come to rest? Gotta be careful about which way static friction will point. But you can ignore rolling friction.



I’ve drawn forces. Now let’s see what the N2L rotation equation says,



Now use relationship ax = -αr to write this in terms of the acceleration of the ball up the plane,



Now let’s use N2L in x direction (up the slope) to figure out *ax*…



So then to figure out when it stops we use:



**Problem 7**. A girl exerts an 85 N force tangent to a solid sphere resting on relatively frictionless water. If the sphere has a mass m = 800kg and radius R = 45cm, how long until it has completed two revolutions?



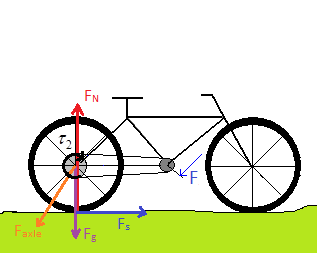
Torque equation…



and now, kinematics equation to get the time…



**Problem 8.** A common example of connected gears is your bike (or a car which has the same kind of set up – minus the chain). Instead of the gears being in physical contact though, they are are connected by a chain – but the result is the same. Suppose the bike pedal has a length L = 12cm. And you step on it with a force of F = 300N perpendicular to the pedal. Suppose the pedal is connected to gear 1 (r1 = 2.5cm), which is connected via the chain to gear 2 (r2 = 5cm). And finally suppose the gear 2 is attached to the wheel, which can be treated as a hoop (mwheel = 10kg, R = 60cm). If the mass of you and the bike is a total of M = 90kg, how fast will you accelerate down the road? Note this problem is very similar to what was done in class. Only difference is the presence of the gears.



First we draw all the forces acting on the wheel (back wheel). The wheel is connected to the axle, and so the axle will exert a force Faxle. We assume the axle is frictionless and so it will not exert a torque. The wheel is connected to gear 2, and so gear 2 will exert a torque on it, τ2. The wheel is in contact with the ground and so there will be normal and friction force on it. Finally the wheel will experience a gravitational force, mwheelg. Now apply N2L rotation to the wheel itself. We’ll treat the wheel as a hoop since it is largely massless in the center. Now to determine the acceleration, first apply N2L translation to the bike + you itself



and now apply the N2L rotation equation to the rear wheel.



Now recognize that these are related via ax = -αR → α = -ax/R. So,



Now solving for Fs in the top equation, we get Fs = Max. And plugging this into the bottom equation we have:



Now what is τ2? This is related to the torque applied at the first gear according to:



Plugging this, and all of the other numbers into our equation we have:

